* $\lim_{x \to +\infty} x^2 (e^{2x} - e^x) = \lim_{x \to +\infty} x^2 e^x (e^x - 1) = +\infty$

* $\lim_{x \to 0} \frac{e^{2x} - 1}{x} =$

$\lim_{x \to 0} \frac{(e^x - 1)}{x}(e^x + 1) = 2$

* $\lim_{x \to 0} \frac{x}{1 - e^{-x}} = \lim_{x \to 0} \frac{xe^x}{e^x - 1} =$

$$\lim_{x \to 0} \frac{e^x}{\frac{e^x - 1}{x}} = \frac{1}{1} = 1$$

* $\lim_{x \to 0^+} \frac{e^x - 1}{\sqrt{2x}} = \lim_{x \to 0^+} \frac{\sqrt{2x}}{2}($

$\frac{e^x - 1}{x}) = 0 \times 1 = 0$

$\lim_{x \to +\infty} x(e^{\frac{1}{x}} - 1) = \lim_{x \to 0^+} \frac{e^x - 1}{x} = 1$

$\lim_{x \to -\infty} x(e^{\frac{1}{x^2}} - 1) = \lim_{t \to 0+} \frac{-1}{\sqrt{t}}(e^t - 1) =$

$\lim_{t \to 0^+} -\sqrt{t}(\frac{e^t - 1}{t}) = 0 \times 1 = 0$ (on pose $t = \frac{1}{x^2}$)

* $\lim_{x \to +\infty} (\frac{x(e^{\frac{1}{x}} - 1)}{e^{\frac{1}{x}} + 1}) = \frac{1}{2}$ d'après ce qui précède

$\lim_{x \to +\infty} x^2 (e^{\frac{1}{x}} - e^{\frac{1}{x+1}}) = ?$

Soit $f(x) = x^2 (e^{\frac{1}{x}} - e^{\frac{1}{x+1}})$

On sait que : $\frac{1}{x} - \frac{1}{x+1} = \frac{1}{x(x+1)}$

On a $f(x) = x^2 e^{\frac{1}{x}} (1 - e^{-\frac{1}{x(x+1)}})$

$$= \frac{x^2 e^{\frac{1}{x}}}{-x(x+1)} (\frac{(1 - e^{\frac{1}{(x+1)x}})}{-\frac{1}{x(x+1)}})$$

Or $\lim_{x \to +\infty} e^{\frac{1}{x}} = 1$ $X = -\frac{1}{x(x+1)}$

$$\lim_{x \to +\infty} \frac{(1 - e^{-\frac{1}{(x+1)x}})}{-\frac{1}{x(x+1)}} =$$

$\lim_{x \to +\infty} -\frac{(e^X - 1)}{X} = -1$

et $\lim_{x \to +\infty} -\frac{x^2}{x(x+1)} = -1$

donc $\lim_{x \to +\infty} f(x) = 1$

de même $\lim_{x \to -\infty} f(x) = 1$

Qarray.tn -Aqra Blech- Qarray.tn